A Labor Market Sorting Model of Scarring and Hysteresis

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August 2023

1 Data construction and sample selection

We rely on two main data sources provided by INPS through the VisitINPS Program: i) data on employment relationships in Italy from 1996 to 2018 (*Uniemens* dataset) and ii) balance sheet data for incorporated Italian firms from 1996 to 2018 (*Cerved* dataset).

Starting from data on virtually the universe of Italian private employment relationships at the contract level, we calculated for each worker their yearly gross real earnings and for the workers with multiple contracts we selected the information associated to the highest paid contract in the year and we find the associated annualized income using information on the number of actual weeks worked. For workers that experience a job flow or are promoted after 2009 we obtain the education level from the *Comunicazioni Obbligatorie* dataset, provided to INPS by the Ministry of Labor, and identify graduates and non-graduates. We label as graduates in the data both workers getting undergraduate education or above, or workers getting specialized diplomas after high-school.

We restrict our focus on workers employed under either full-time or part-time working schemes between 16 and 65 years old. Moreover, in order to compute AKM fixed effects and maintain the estimation computationally feasible, we analyze the connected set of firms and workers across firms with more than 15 employees. As in Stephane Bonhomme, Thibaut Lamadon and Elena Manresa (2019) we first cluster firms by means of a weighted K-means clustering based on deciles of their wage distribution. The weight for the clustering is the yearly number of employees.

From *Cerved*, we have access to firm balance sheet data. From this dataset we obtain information of value added, overall labor costs and calculate quantiles of value added per employee. All monetary values (wages, value added, total compensation) are trimmed at the 1% level and deflated by the consumer price index for Italy.

2 Derivation of recursive contract SPFE

Solving the optimal contract and the overall model given the recursive structure obtained by following the promised utility method of Stephen E. Spear and Sanjay Srivastava (1987) is computationally infeasible. This is due to the fact that the optimal contract requires to define a valid recursive domain and codomain of promised values that respects all the future forward looking constraints. Known solution methods for these kinds of models (Dilip Abreu, David Pearce and Ennio Stacchetti, 1990), although robust, easily become computationally unmanageable as the number of states of the model increases. We thus follow Albert Marcet and Ramon Marimon (2019) (hereby MM) in deriving a recursive expression for the optimal contract in which the Lagrange multiplier for the promise keeping constraint **Equation 4** is added as a co-state of the model, and allows us to circumvent the problem of searching for valid promised values domains altogether. Wage growth and levels in any next period and at every node are then determined by the state-contingent multiplier on tomorrow's promise keeping constraints. This considerably reduces the complexity of the problem, as by definition Lagrange multipliers are defined over $\mathbb{R}^+$.

We adopt MM's terminology to define how a recursive saddle point functional equation (SPFE) can be obtained from the sequential formulation of the problem. For the present exposition of the constructive method to obtain the SPFE, for simplicity and without loss of generality, we ignore the randomization of the contract over the lotteries and the limited liability constraint. The latter choice, in particular, does not create any problem in terms of thinking about of developing the sequential problem over time: our choice of timing of exit decision is such as that exiting firms know form the start of their period whether the productivity level is below the critical one $a^*_{h,\tau,\iota,y}$ for the match (h, τ, ι, W, y), and thus whether they will exit or not. The lack of uncertainty and optimization over the next periods makes the problem of these firms, at some low states, equivalent to the problem of a firm with a lower maximum length (which is T, the retirement age, in general). At an exiting state t the firm knows *with certainty* that any $J_j = 0$ for $j > t$, match with a worker of age T.

Proposition 2.1. *The solution to the contracting problem in **Equation** (5) is the same as the solution to the following saddle-point functional equation:*

$$\begin{aligned}\mathcal{P}_t(h_t, \tau_t, y_t, a_t, \gamma_t) =& \underset{\gamma_t}{inf}\,\underset{w_t}{sup}(f(a_t, y_t, h_t) - w_t) + \mu^1_t W_{y,t} - \gamma^1_t(W_{y,t} - u(w_t)) + \\ & \beta\mathbb{E}_t(\lambda U_{t+1} + (1-\lambda)\lambda_e p_{t+1} v^*_{t+1}) + \beta\mathbb{E}_t\widetilde{p}_{t+1}\mathcal{P}_{t+1}(h_{t+1}, \tau_{t+1}, y_{t+1}, a_{t+1}, \gamma_{t+1})\end{aligned}$$

with $\mu_t = \gamma_{t_1}$ for some starting γ_0.

Proof. Consider the problem[1]

$$J_t(h_t, \tau_t, y_t, W_t, a_t) = \sup_{w_t, \{W_{s^{t+1}}\}} \Big(f(a_t, y_t, h_t) - w_t \\ + \mathbb{E}_{s^t} \left[\widetilde{p}(h_{t+1}, \tau_{t+1}, W_{s^{t+1}}, a_{s^{t+1}})(J_{t+1}(h_{t+1}, \tau_{t+1}, y_t + 1, W_{s^{t+1}}, a_{s^{t+1}})\right] \Big) \tag{1}$$

$$s.t.\ W_t = u(w_t) + \beta \mathbb{E}_{s^t} \Big(\lambda U_t(h_{t+1}, \tau_{t+1}, a_{t+1}) + \\ (1-\lambda)(\lambda_e p_{t+1}(h_{t+1}, \tau_{t+1}, W_{s^{t+1}}, a_{s^{t+1}}) v^*(h_{t+1}, \tau_{t+1}, W_{s^{t+1}}, a_{s^{t+1}}) \\ + (1 - \lambda_e p_{t+1}(h_{t+1}, \tau_{t+1}, W_{s^{t+1}}, a_{s^{t+1}})) W_{s^{t+1}}) \Big) \tag{2}$$

We define as endogenous states $\mathbf{x}_t = [h_t, \tau_t, y_t, W_t]$, controls $\mathbf{c}_t = [w_t, W_{s^{t+1}}]\ \forall t, s^{t+1}$, whereas the only exogenous state is a_t. The endogenous states follow the law of motion

$$\mathbf{x}_{t+1} = \begin{bmatrix} h_{t+1} \\ \tau_{t+1} \\ y_{t+1} \\ W_{s^{t+1}} \end{bmatrix} = l(\mathbf{x}_t, \mathbf{c}_t, a_{s^{t+1}}) = \begin{bmatrix} \phi(h_t, y_t) \\ \tau_t + 1 \\ y_t \\ W_{s^{t+1}} \end{bmatrix} \tag{3}$$

In the subsequent notation, where appropriate, we omit listing all states on which elements in the equation, and subsume their dependence under just listing the time t. J can be rewritten, by developing forward the recursion until time T, at which the match surely dissolves, as

$$J_t(\{h_t, \tau_t, y_t, W_t, a_t\}_{t=t_0}^{T-t_0}) = \mathbb{E}_{t_0} \sum_{t=t_0}^{T} \beta^{t-t_0} \prod_{i=0}^{t-t_0} \widetilde{p}_{t_o+i} \Big(f(a_t, y_t, h_t) - w_t \Big) \tag{4}$$

where $\widetilde{p}_{t_0} = 1$. Notice that the forward-looking constraint in **Equation 2** is state contingent and an instance of it applies at *every* node of any possible history $s^t\ \forall t$ given the prevailing W_y promised at that node. The equilibrium is an instance of subgame perfect Nash equilibrium in which an agent chooses its strategies while anticipating the best response of the following agent, as common in dynamic games with a leader-follower component introduced by Heinrich Von Stackelberg (1934). The structure of the problem and the solution also shares some commonality with Ramsey optimal policy problems in which a policy maker (in this case the firm) optimizes the utility of all agents according to some weights and taking into account their optimal behavior.[2]

[1] Without loss of generality, we ignore the level of education ι in the proof, as it is a fixed worker characteristic.

[2] In the terminology of MM, we treat constraints coming from **Equation 2** as a set of one period ahead forward looking constraint, which makes the analysis of our case akin to their case where one have

We can redefine the problem:

$$V_{t_0}(\mathbf{x}_t, a_t) = \underset{\{w_{s^t}, W_{s^t}\}}{sup} \mathbb{E}_{t_0} \sum_{t=t_0}^{T} \beta^{t-t_0} \prod_{i=0}^{t-t_0} \widetilde{p}_{t_o+i}\Big(f(a_t, y_t, h_t) - w_t\Big) \tag{5}$$

$$s.t.\ [j=0]: \sum_{t=t_0}^{T} \beta^{t-t_0} \prod_{i=0}^{t-t_0} \widetilde{p}_{t_o+i}\Big(f(a_t, y_t, h_t) - w_t\Big) - R \geq 0 \tag{6}$$

$$[j=1, s^t]: \ W_{s^t} - u(w_{s^t}) - \beta \mathbb{E}_{s^t}\Big(\lambda U_{s^{t+1}} + (1-\lambda)(\lambda_e p_{s^{t+1}} v^*_{s^{t+1}}) + \widetilde{p}_{s^{t+1}} W_{s^{t+1}})\Big) \geq 0 \tag{7}$$

where the constraint 13 is a slack participation constraint for a sufficiently small R, so that the principal (the firm) is willing to enter the contract in the first place.

In the terminology of MM we can label

$$h_0^0(\mathbf{x}_t, \mathbf{c}_t, a_t) = f(a_t, y_t, h_t) - w_t \tag{8}$$

$$h_1^0(\mathbf{x}_t, \mathbf{c}_t, a_t) = f(a_t, y_t, h_t) - w_t - R \tag{9}$$

$$h_0^1(\mathbf{x}_t, \mathbf{c}_t, a_t) = W_t \tag{10}$$

$$h_1^1(\mathbf{x}_t, \mathbf{c}_t, a_t) = W_t - u(w_t) + \beta \mathbb{E}_t(\lambda U_{t+1} + (1-\lambda)\lambda_e p_{t+1} v^*_{y,t+1}) \tag{11}$$

and define the Pareto problem ($\mathbf{PP}_\mu$)

$$\mathbf{PP}_\mu : V_{\mu,t_0}(\mathbf{x}_t, a_t) = \underset{\{w_{s^t}, W_{s^t}\}}{sup} \mathbb{E}_{t_0} \sum_{t=t_0}^{T} \beta^{t-t_0} \prod_{i=0}^{t-t_0} \widetilde{p}_{t_0+i} \mu^0 \Big(f(a_t, y_t, h_t) - w_t\Big) + \mu^1 W_{t_0} \tag{12}$$

$$s.t.\ [j=0;\ \gamma^0]: \sum_{t=t_0}^{T} \beta^{t-t_0} \prod_{i=0}^{t-t_0} \widetilde{p}_{t_o+i}\Big(f(a_t, y_t, h_t) - w_t\Big) - R \geq 0 \tag{13}$$

$$[j=1, s^t;\ \gamma^1_{s^t}]: \ W_{s^t} - u(w_{s^t}) - \beta \mathbb{E}_{s^t}\Big(\lambda U_{s^{t+1}} + (1-\lambda)(\lambda_e p_{s^{t+1}} v^*_{s^{t+1}}) + \widetilde{p}_{s^{t+1}} W_{s^{t+1}})\Big) \geq 0 \tag{14}$$

Still following the notation from Marcet and Marimon (2019), we can define the Saddle

$j = 1$ forward looking constraints, and $N_1 = 0$. The difference with their problems, however, is that our problem features finite time, and thus each one period ahead forward looking constraint technically applies to a *different* function j_t (indexed by t).

Point Problem ($\mathbf{SPP}_\mu$) as:

$$\mathbf{SPP}_\mu : SV_{\mu,t_0}(\mathbf{x}_{t_0}, a_{t_0}) = \underset{\{\gamma \in \mathbb{R}^l_+\}}{inf} \underset{\{w_{s^t}, W_{s^{t_0}}\}}{sup} \mu^0 \Big(f(a_{t_0}, y_{t_0}, h_{t_0}) - w_{t_0} \Big) + \mu^1 W_{t_0} +$$
$$+ \beta \mathbb{E}_t \left(\phi(\mu, \gamma) \sum_{i=0}^{T-t_0} \left[\beta^{t_0+i} \prod_{i=0}^{T-t_0-1} \widetilde{p}_{t_0+1+i} \left(f(a_{t_0+i}, y_{t_0+i}, h_{t_0+i}) - w_{t_0+i} \right) + W_{t_0+i} \right] \right) +$$
$$+ \gamma^1 \left(u(w_{t_0} + \beta \mathbb{E}_{t_0} \left(\lambda U_{t_0+1} + (1-\lambda) \lambda_e p_{t_0+1} v^*_{y,t_0+1} \right) \right) +$$
$$+ \gamma^0 \left(f(a_{t_0}, y_{t_0}, h_{t_0}) - w_{t_0} - R \right) \tag{15}$$

The problem can be restated as a saddle-point problem over a Lagrangian equation

$$\underset{\gamma_t}{inf} \underset{\{w_{s^t}, W_{s^t}\}}{sup} \mu^0 \Big(f(a_{t_0}, y_{t_0}, h_{t_0}) - w_{t_0} \Big) + \mu^1 W_{t_0} +$$
$$\gamma^0 \Big((f(a_{t_0}, y_{t_0}, h_{t_0}) - w_{t_0} \Big) - R) +$$
$$\gamma^1_{t_0} \Big(- W_{t_0} + u(w_{t_0}) + \beta \mathbb{E}_{t_0} (\lambda U_{t_0+1} + (1-\lambda)(\lambda_e p_{t_0+1} v^*_{t_0+1} + \widetilde{p}_{t_0+1} W_{t_0+1}) \Big) +$$
$$\beta \mathbb{E}_{t_0} \Bigg[(\mu_0 + \gamma_0) \sum_{t=t_0+1}^{T} \beta^{t-t_0-1} \prod_{i=0}^{T-t_0-1} \widetilde{p}_{t_0+1+i} \Big(f(a_t, y_t, h_t) - w_t \Big) +$$
$$\sum_{t=t_0+1}^{T} \mathbb{E}_t \beta^{t-t_0-1} \prod_{i=0}^{t-t_0-1} \widetilde{p}_{t_0+1+i} \gamma^1_t \Big(- W_t + u(w_t) +$$
$$\beta(\lambda U_{t+1} + (1-\lambda)(\lambda_e p_{t+1} v^*_{t+1} + \widetilde{p}_{t+1} W_{t+1}) \Big) \Bigg] \tag{16}$$

which, thanks to some algebra and the law of iterated expectations becomes

$$\underset{\gamma_t}{inf} \underset{\{w_{s^t}, W_{s^t}\}}{sup} - \gamma^0 R + \mathbb{E}_{t_0} \sum_{t=t_0}^{T} \beta^{t-t_0} \prod_{i=0}^{t-t_0} \widetilde{p}_{t_0+i} \Bigg[\Big(\mu^0_t + \gamma^0_t \Big) \Big(f(a_t, y_t, h_t) - w_t \Big) + \mu^1_t W_t -$$
$$\gamma^1_t \Big(W_t - u(w_t) - \beta(\lambda U_{t+1} - (1-\lambda) \lambda_e p_{t+1} v^*_{t+1} \Big) \Bigg] \tag{17}$$

where $\mu^0_t = \mu^0 = 1$, $\gamma^0_t = \gamma_0 = 0$, $\mu^1_t = \gamma^1_{t-1}$ for some starting $\gamma^1_{t_0-1}$.

The problem can now be written in recursive form. Define

$$\mathcal{P}_t(h_t, \tau_t, y_t, a_t, \gamma_t) = \underset{W_t}{sup}\, J_t(h_t, \tau_t, y_t, W_t, a_t) + \mu^1_t W_t \tag{18}$$

Given **Equation 17** the SPFE of the problem can be written as

$$\mathcal{P}_t(h_t,\tau_t,y_t,a_t,\gamma_t) = \inf_{\gamma_t} \sup_{w_t} (f(a_t,y_t,h_t) - w_t) + \mu_t^1 W_t - \gamma_t(W_t - u(w_t)) + \beta \mathbb{E}_t(\lambda U_{t+1} + (1-\lambda)\lambda_e p_{t+1} v_{t+1}^*) + \beta \mathbb{E}_t \widetilde{p}_{t+1} \mathcal{P}_{t+1}(h_{t+1},\tau_{t+1},y_{t+1},a_{t+1},\gamma_{t+1}) \tag{19}$$

One can easily verify that the solution of this equation is the same we found in the maximization of **Equation 5** in the main text. Take the first order conditions and compute the envelope condition:

$$[FOC\ w_t]: \ -1 + \gamma_t u'(w_t) = 0 \tag{20}$$

$$[ENV\ W_t]: \ \frac{\partial \mathcal{P}_t}{\partial W_t} = \mu_t^1 - \gamma_t \tag{21}$$

$$[FOC\ W_{t+1}]: \ -\widetilde{p}_{t+1} W_{t+1} \gamma_t + \frac{\partial \widetilde{p}_{t+1}}{\partial W_{t+1}} \mathcal{P}_{t+1} + \widetilde{p}_{t+1} \frac{\partial \mathcal{P}_{t+1}}{\partial W_{t+1}} = 0 \tag{22}$$

where **Equation22** is obtained by adding and subtracting from **Equation 19** $\beta\gamma_t\widetilde{p}_{t+1}W_{t+1}$. The reader should also keep in mind that the condition in **Equation 22** is actually state contingent and applied to *all* future states next period, with a different set of co-states $\gamma_{s^{t+1}}$ for each realization of a_{t+1}.

Some rearranging of the **Equation 22** leads to the following result

$$\frac{\partial \log \widetilde{p}_{t+1}}{\partial W_{t+1}} \bigg(\mathcal{P}_{t+1} - \gamma_t W_{t+1} \bigg) = \gamma_{t+1} - \mu_{t+1}^1 \tag{23}$$

which, given the law of motion of the co-states and the definition in **Equation 18** can be re-written as:

$$\frac{\partial \log \widetilde{p}_{t+1}}{\partial W_{t+1}} J_{t+1} = \frac{1}{u'(w_{t+1})} - \frac{1}{u'(w_t)} \tag{24}$$

which is exactly **Equation 14**, namely the Euler equation that governs the behavior of wage setting and disciplines the provision of insurance within the contract. □

3 Model solution and estimation

Model solution. For each education level, we solve the model by backward induction on a regular grid of human capital, wage multiplier levels, firm quality and aggregate shocks. In particular, starting from the last period of workers lives we compute the terminal wage in each state, then solve the search problems for both the employed and the unemployed, which allows us to compute the market tightness in each submarket as well as the value of the contract and of unemployment. With these objects in hand we proceed backwards until workers labor market entry.

In the model simulations, we populate each cohort of agents with 42 individuals, 30 non-graduates and 12 graduates. Graduates enter the labor market in a staggered manner every quarter for the first three years of their life. We then simulate the model for 600 periods (we take 180 periods as burn in) and construct a panel with 419 periods and $7,560$ individuals per period. In the model, we consider one period as one quarter in the data.[3]

Estimation. To pin down the 18 internal parameters in the model, $\boldsymbol{\theta}$, we minimize the absolute error between model simulated moments, $m(\boldsymbol{\theta})$, and their empirical counterparts, $\mathbf{d}$. More formally, we pick the vector of parameters

$$\boldsymbol{\theta}^* = \arg\min_{\boldsymbol{\theta}} \left\{ (|m(\boldsymbol{\theta}) - \mathbf{d}|)' W (|m(\boldsymbol{\theta}) - \mathbf{d}|) \right\}.$$

In $\mathbf{d}$ and $m(\boldsymbol{\theta})$ we stack: the profiles of E-E and E-U transitions by age (9 age groups); wage growth relative to the initial wage by education level and experience (9 experience groups); the unemployment rate; the correlations of labor market flows with the cyclical component of aggregate output; the average inactivity rate; average sorting, measured as the average correlation between AKM fixed effects in the data and as the correlation of worker and firm qualities in the model; the ratio of initial wages between graduates and non-graduates, and the ratios of the second and third quintile of value added per employee to the first.

Jointly, these ten moments deliver 44 restrictions for 18 parameters. We construct each moment in the model weighting by age from Italian population weights. In the SMM we weight moments so that each age profile, as a whole, has the same weight of the other moments and, within each profile, we give more weight to age groups that have a higher demographic share in the Italian population.[4] We collect this weighting scheme in the matrix W.

We solve for $\boldsymbol{\theta}^*$ numerically adopting two complementary strategies. In the first, we use a two step procedure and minimize the distance between model-generated and data-generated moments using a global solver on the largest feasible domain, for this,

[3]We use a grid of $20 \times 15 \times 25 \times 7$ for each education level and we consider 180 quarters of workers life. Solving and simulating the model takes approximately 25 minutes with Python 3.11 on a Linux HPC cluster with 52 cores and 120G of RAM.

[4]For example, the weight for the EE rate of 18-22 years old has a weight equal to $1/10 \times 0.091$, the share of 18-22 years old in the Italian population, while the weight on the unemployment rate instead is $1/10$, so that the entire EE profile, as a whole, weights as much as the unemployment rate.

we rely on the particle swarm algorithm contained in the library developed by (Julian Blank and Kalyanmoy Deb, 2020). In the second, we solve and simulate the model over a sparse grid of the parameter space.[5] In both cases, we refine the estimation using the best combination of parameters as the starting point of a local minimization routine, for this we rely on the Nelder-Mead algorithm in the SciPy library. **Table III** in the paper reports the averages for the profiles and the other targets, **Figure 2** plots the whole profiles.

4 State dependence

We check for state dependence by running the following regression across different model simulations

$$Y_{\text{Post}} = \alpha + \sum_{j=1}^{4} \beta_j \underbrace{\mathbb{E}[FQ^j_{\text{Pre}}]}_{\text{Firm Quality}} + \gamma_j \overbrace{\mathbb{E}[HC^j_{\text{Pre}}]}^{\text{Human Capital}} + \delta_j \underbrace{\mathbb{E}[W^j_{\text{Pre}}]}_{\text{Wages}} + \eta_j \overbrace{\mathbb{E}[LS^j_{\text{Pre}}]}^{\text{Labor Share}} + \theta \underbrace{\mathbb{C}(FQ_{\text{Pre}}, HC_{\text{Pre}})}_{\text{Sorting}} + \varepsilon,$$

in which $\mathbb{E}[X^j_{\text{Pre}}]$, denotes the j^{th} moments of an endogenous, cross-sectional distribution before the shock, and Y_{Post} are model outcomes after the shock. Results are shown in **Table 1** and **Table VI** in the main body.

5 Additional Figures and Tables

[5] The advantage of selecting the parameters with this procedure is that the grid is built so that the function domain is optimally covered with the least amount of possible points compared to other forms of approximation (e.g. equi-spaced or random grids). The sparse grid is built using the"Tasmanian" libraries (M Stoyanov, 2015).

Table 1. State Dependence

(a) Mean

	Output Response		Persistence
	Short-term	Medium-term	
Firm Quality	**-1.6*****	**-2.1****	-0.5
	(0.6)	(1.0)	(1.0)
Human Capital	**2.1****	1.6	**2.6***
	(0.8)	(1.4)	(1.4)
Wage	0.1	0.3	-0.0
	(0.4)	(0.6)	(0.6)
Labor Share	**-2.4*****	**-4.1*****	**1.8***
	(0.6)	(1.1)	(1.1)
Sorting	**2.1****	**3.4****	0.9
	(1.0)	(1.7)	(1.6)
R^2	0.6	0.5	0.1
N	100	100	100

(b) Variance

	Output Response		Persistence
	Short-term	Medium-term	
Firm Quality	**0.2***	**0.3***	0.1
	(0.1)	(0.2)	(0.2)
Human Capital	**-1.1***	**-1.4***	-1.2
	(0.5)	(0.8)	(0.8)
Wage	-0.2	-0.3	0.0
	(0.2)	(0.4)	(0.4)
Labor Share	-10.2	-6.9	-18.6
	(9.2)	(15.8)	(15.3)
R^2	0.6	0.5	0.1
N	100	100	100

(c) Skewness

	Output Response		Persistence
	Short-term	Medium-term	
Firm Quality	**-0.6****	**-0.9***	-0.2
	(0.3)	(0.5)	(0.5)
Human Capital	1.2	0.3	**2.6***
	(0.9)	(1.6)	(1.5)
Wage	**-0.4****	-0.3	**-0.7****
	(0.2)	(0.3)	(0.3)
Labor Share	0.3	0.3	**0.9***
	(0.3)	(0.5)	(0.5)
R^2	0.6	0.5	0.1
N	100	100	100

(d) Kurtosis

	Output Response		Persistence
	Short-term	Medium-term	
Firm Quality	**0.05****	**0.1***	0.0
	(0.03)	(0.04)	(0.0)
Human Capital	-0.2	-0.1	-0.4
	(0.1)	(0.2)	(0.2)
Wage	0.0	0.0	**0.1****
	(0.0)	(0.1)	(0.1)
Labor Share	0.2	0.2	-0.1
	(0.2)	(0.4)	(0.4)
R^2	0.6	0.5	0.1
N	100	100	100

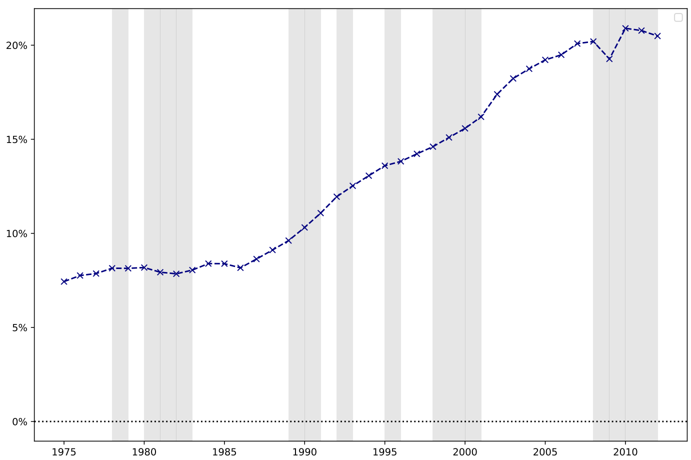
Note: Standard-errors in parenthesis. p-value: $^{*} < 0.1$, $^{**} < 0.05$, $^{***} < 0.01$. The table reports the coefficients of regressing model outcomes after the shock on moments of relevant endogenous variables before the shock (one year average). The results are scaled to report the effect of changing the regressors by 1pp. Sorting is computed every quarter as the correlation between firm and worker quality and reported in the same table of the means even if technically is not a moment of a distribution. Model outcomes are: i) short-term cumulative output response (1 year after the shock); ii) the medium-term cumulative output response (3 years after the shock); and iii) the persistence of the shock (number of quarters before the output IRF is back at zero). For ease of exposition, the results are grouped by moments but the coefficients are computed including all moments in the same regression. The simulations are those underlying **Figure 6**.

Table 2. Effects of initial aggregate conditions along the experience profile and experience growth profile

Dep.Variable: Log-Wage	Experience × Cycle	Experience
Experience Dummy		
0	1.968	
	(0.452)	
1	1.463	0.150
	(0.238)	(0.011)
2	1.473	0.246
	(0.283)	(0.014)
3	1.239	0.304
	(0.343)	(0.014)
4	1.250	0.342
	(0.349)	(0.015)
5	1.239	0.375
	(0.349)	(0.015)
6	1.301	0.399
	(0.287)	(0.015)
Age FE	✓	✓
Year FE	✓	✓
Sex FE	✓	✓
LLM FE	✓	✓
R^2	0.89	0.89
N	254,000,000	254,000,000

Note: The table reports regression coefficients for the empirical estimates in the data used to construct the profiles in Figure D.2.

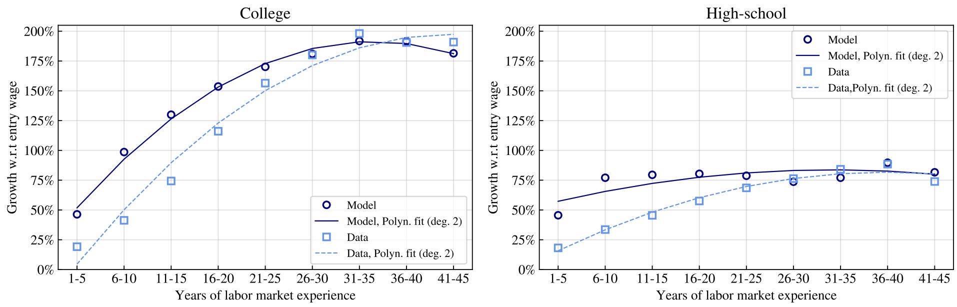
Figure 1. Tertiary School Enrollment and the Business Cycle



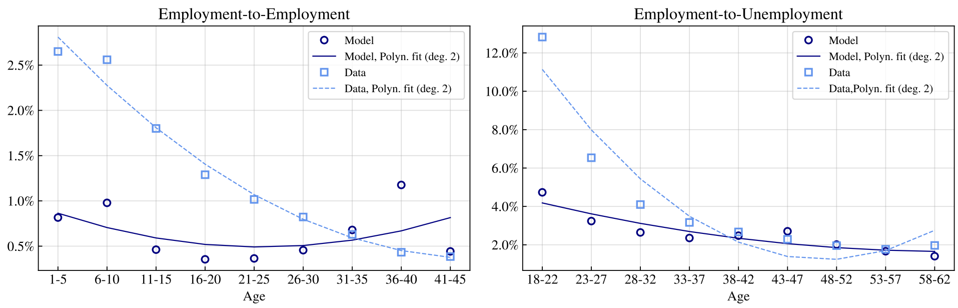
Note: The figure plots the ratio of students enrolled in tertiary education to population aged 16-29 in every year. Source: Italian National Institute of Statistics (ISTAT). Shaded areas indicate the OECD based Recession Indicators for Italy.

Figure 2. Target profiles

(a) Wage Profiles



(b) Transition rates



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